Ill-Posedness, Chaotic Dynamics, and Discontinuous Parameterizations in Geophysical and Climate Science Models: An Extended Critical Survey

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The Unifying Mathematical Signature and Critical Principles

The examples in this survey share the same mathematical fingerprint: a system of first-order partial differential equations (PDEs) whose linearized coefficient matrix has complex characteristics in some parameter regime, or whose initial-boundary value problem is ill-posed in the sense of Hadamard .

Hadamard ill-posedness means that no finite constant C exists such that the growth rate $\sigma(k) \leq C$ for all wavenumbers k. Instead, $\sigma(k) \to \infty$ as $k \to \infty$. Grid refinement invites faster growth, making convergence impossible by construction.

Critical principle. When ill-posedness is identified, the mathematically and physically correct response is to identify the genuine physical process that provides the missing stabilizing mechanism and to include it from first principles. Adding artificial dissipation, drag, sponge layers, or flux limiters solely to achieve numerical stability — without physical justification from the fundamental equations — suppresses a real mathematical pathology without curing it, and introduces a false length or time scale that contaminates all predictions at that scale. This distinction between genuine physical regularization and what the two-phase thermal-hydraulics community has called “regularization by illegal addition” is the central concern of this survey.

Practical Ramifications. The Lax Equivalence Theorem states that *convergence* of numerical solutions of discrete approximations to solutions of the continuous equations is ensured when (1) the finite difference approximations are *consistent with* the continuous equations, and (2) the numerical solution method is *stable*. The presence of complex characteristics ensures that stable numerical solution methods for consistent discrete approximations are not possible.

These aspects of ill-posed equation systems are summarized in the paper by Lyczkowski *et al.* (1978), in which citations to the basic literature are given. The papers by Gidaspow

(1974) and Gidaspow et al. (1973) provide additional information about the fundamental problems.

Various regularizations are frequently used in attempts to rectify the outcome of ill-posed equation systems. These will sometimes make the finite approximations differ from a consistent approximation. Convergence to solutions of the continuous equations is not ensured.

Example 1: The Hydrostatic Approximation in GCM Dynamical Cores

The ill-posedness result of Oliger and Sundström (1978)

The most consequential and least-acknowledged ill-posedness in all of geophysical modeling affects the dynamical core of virtually every operational global and regional climate model.

Oliger and Sundström (1978) proved rigorously in the *SIAM Journal on Applied Mathematics* that **the hydrostatic primitive equations of meteorology and oceanography are ill-posed with any specification of local, pointwise boundary conditions**. The mechanism is as follows. The vertical normal mode expansion of the hydrostatic equations produces a spectrum of modes with characteristic speeds $c_n^{\pm} = U_0 \pm N/\lambda_n$, where U_0 is the background wind, N is the Brunt-Väisälä frequency, and λ_n is the n-th vertical eigenvalue. Subcritical modes ($|U_0| < N/\lambda_n$) require two boundary conditions on one lateral face and one on the other; supercritical modes ($|U_0| > N/\lambda_n$) require three on one face. Since any given flow will have a mixture of subcritical and supercritical modes, and since no fixed set of local (pointwise) boundary conditions can simultaneously satisfy all modes, the system is ill-posed .

This result applies to:

- All regional NWP and regional climate models (WRF, RegCM, COSMO, ALADIN, HARMONIE, MAR) that use the hydrostatic primitive equations on limited-area domains.
- All global GCMs that use domain decomposition with lateral interface conditions for parallel computation.
- All one-way nested regional climate downscaling simulations.

The Browning-Kreiss analysis (1986, 2002)

Browning and Kreiss (1986, 2002) showed that the assumption of *exact* hydrostatic balance introduces multiple mathematical problems by over-constraining the system. The vertical velocity is determined diagnostically from the horizontal divergence (Richardson's equation) rather than evolving prognostically, and the resulting columnar (one-dimensional in the vertical) equation for w is not the correct three-dimensional quantity. Consequences include:

1. Loss of hyperbolicity for lateral boundary value problems .

2. Inaccurate representation of vertically propagating gravity waves, with the hydrostatic solution diverging from the true atmospheric solution by $\sim 50\%$ within 36 hours under some conditions.
3. The apparent success of hydrostatic weather prediction is partly due to nonlinear normal-mode initialization, which suppresses fast time scales — hiding the ill-posedness rather than curing it.

Browning and Kreiss (2002) introduced a mathematically correct alternative: rather than removing vertical acoustic waves entirely (exact hydrostatic balance), they proposed slowing those waves down, retaining full hyperbolicity. This was proved to accurately describe large-scale atmospheric motions .

Is the hydrostatic ill-posedness ever principally corrected?

In global GCMs on the closed sphere with fully periodic boundary conditions the Oliger-Sundström lateral boundary ill-posedness does not arise in its precise form. However the Browning-Kreiss defect remains: the system is not strictly hyperbolic in 3-D and its solutions are not asymptotically accurate approximations to the true Navier-Stokes solutions at convective, gravity wave, and orographic scales.

In regional models and limited-area domains the full Oliger-Sundström ill-posedness is directly present. Every operational regional climate model uses one of the following engineering workarounds:

1. *Artificial vertical viscosity* (a δw drag term in the hydrostatic equation) : restores formal well-posedness but introduces a physically unjustified dissipation coefficient δ with no derivation from the governing equations.
2. *Sponge layers* at lateral boundaries: all fields relaxed toward prescribed values within several grid cells of the boundary. Used in WRF, RegCM, COSMO, ALADIN, and all other operational RCMs. The sponge width and relaxation coefficient are free parameters, not derived from physical principles.
3. *Relaxation nudging* toward larger-scale model fields: standard in all one-way nested RCMs.
4. *Non-local (mode-by-mode) boundary conditions* : the only mathematically principled fix. Computationally expensive and not used in any operational model.

Approaches (1)–(3) constitute regularization by illegal addition. They hide the mathematical pathology without curing it and introduce false scales whose effects on projected climate variables have never been assessed.

In non-hydrostatic models (WRF-NMM, ICON, IFS-NH, UM non-hydrostatic, NICAM, MPAS) strict hyperbolicity is restored, subject to proper treatment of acoustic modes. However, most current global climate models (CESM/CAM, GFDL-ESM4, IPSL-CM6, MPI-ESM, ACCESS-CM2, CNRM-CM6) still use hydrostatic dynamical cores and will do so for decades given the computational cost of non-hydrostatic global simulation at climate resolution.

Vertical discretization ill-posedness

Bourchtein (2009) showed in the *Quarterly Journal of the Royal Meteorological Society* that **non-local vertical discretization schemes in hydrostatic models can themselves introduce ill-posed initial value problems**, independent of the lateral boundary question. Standard local vertical differencing is provably well-posed; greater non-locality in the vertical scheme is not. This is a purely numerical ill-posedness invisible to users examining model outputs.

Example 2: The Historical Basis in Two-Phase Thermal Hydraulics

The rigorous identification of Hadamard ill-posedness in coupled multiphase flow systems was carried out within the nuclear reactor thermal-hydraulics community, beginning with the foundational analyses of Gidaspow, Lyczkowski, Solbrig, and Hughes, and Ramshaw and Trapp at Idaho National Laboratory, in the period 1973–1978.

Gidaspow, Lyczkowski, Solbrig, and Hughes (1973–1980)

Gidaspow, Lyczkowski, Solbrig, and Hughes presented the first identification of complex characteristics in standard two-phase flow equation systems at the American Nuclear Society in 1973, and expanded this at the 5th International Heat Transfer Conference (1975). The full characteristics and stability analysis, including finite-difference approximations, was published in *Nuclear Science and Engineering* (1978), demonstrating conclusively that: (1) standard single-pressure two-fluid models have complex characteristics under separated and dispersed flow conditions; (2) resulting Hadamard growth rates are proportional to wavenumber; and (3) coarse-grid finite differences mask the instability through numerical diffusion. Hirt (1968) and Ramshaw (1994) explicitly developed the source of these numerical diffusion terms for several finite difference approximations.

Lyczkowski (1980) subsequently proved analytically why single-pressure two-fluid unequal-velocity models are not globally hyperbolic, identifying the specific pressure-exchange terms responsible for complex characteristics.

Lyczkowski (2018) provides the authoritative history of this entire field in his memoir *The History of Multiphase Science and Computational Fluid Dynamics*.

While this research is frequently cited to be the initial discovery of ill-posedness in the multi-phase, thermal-hydrodynamics engineering literature, the matter seems to have been first identified by Rakhmatulin (1956). He subsequently published, Rakhmatulin (1969), a report on sound speeds for two-phase mixture theories in which the case of non-equilibrium mixture theory is explicitly omitted. Clearly indicating the lack of real characteristics.

Ramshaw and Trapp (1976, 1978)

Ramshaw and Trapp at Idaho National Engineering Laboratory performed an independent analysis of characteristics, stability, and short-wavelength phenomena in two-phase flow equations. Their work made explicit the connection between complex characteristics, Hadamard instability, and the non-uniform convergence of finite-difference schemes: a coarser grid may appear stable while a finer grid blows up, precisely because the finer grid resolves the ill-posed short-wavelength modes.

The analyses show clearly that the several physical instabilities, Raleigh, Taylor, Kevin, Helmholtz, among others all described by model equation systems having real characteristics, nonetheless can exhibit growth usually given by some critical values of state space dependent variables. Linear dispersion analyses of the equation systems are generally employed to determine these critical values. Thus, while real characteristics of the basic equations, which account for only derivative terms, are necessary, that analysis is not sufficient to determine the presence of potential growth of perturbations.

The Drew-Segel averaging framework (1971–1998)

Hundreds of derivations of multi-phase model equation systems have appeared in the literature starting with the research results at the Idaho National Laboratory in the early 1970s. Far too many, by far too many different approaches to summarize in these notes. The papers by Drew and coworkers (1971, 1983, 1998) are among the more assessable. And early review was given by Beford and Drumheller (1983). The averaging process that produces two-fluid equations from microscale Navier-Stokes necessarily introduces closure terms whose mathematical structure sometimes rely on differential terms— and therefore whether the resulting system is hyperbolic — depends entirely on the closure model chosen.

Stewart and Wendroff (1984)

Stewart and Wendroff (1984) provided a comprehensive review in *Journal of Computational Physics*, cataloguing the characteristic structure of numerous two-phase model variants and proving hyperbolicity conditions for each. They demonstrated that the standard single-pressure models used in reactor safety codes TRAC, RELAP, and RETRAN are ill-posed for physically relevant conditions.

Other Geophysical Examples

Subglacial hydrology

The SHAKTI distributed subglacial hydrology model , without lateral heat diffusion, has a thermal melt feedback instability with $\sigma(k) \to \infty$. showed that lateral heat diffusion — a genuine physical process in ice — regularizes the system and predicts observed channel spacing. This is one of the very few cases in the geophysical literature where the correct physical regularization was identified and added on physical grounds.

River morphodynamics (Hirano active layer)

The Hirano (1971) mixed-sediment active layer model is ill-posed under armoring conditions . The model remains default in Delft3D, MIKE, and TELEMAC-MASCARET despite published well-posed alternatives.

Dense granular flows ($\mu(I)$-rheology)

Baker and coworkers (2015, 2016, 2017) proved the $\mu(I)$-rheology is ill-posed at both slow and fast flow limits; and provide partial physical regularizations via smooth $\mu(I)$ and compressibility.

Depth-averaged debris flow models

Coulomb-rheology models are always ill-posed ; bidisperse segregation models are ill-posed above a critical velocity-size ratio . The D-Claw model was designed explicitly as a hyperbolic system.

Two-layer shallow water stratification

The classical two-layer shallow water equations have complex characteristics in certain regimes . This affects ocean layer models, estuarine dynamics, and atmospheric density current models.

Ice sheet grounding line

The thin-film approximation near the grounding line produces grid-dependent flux singularities , contributing to decades of non-converging ice sheet projections in CMIP ensembles.

Aeolian dune migration

Local saltation flux models without a saturation length produce $\sigma \propto k$. Planetary dune models for Mars and Titan that omit L_{sat} produce mesh-dependent results.

The Three Critical Systemic Questions

Is illegal regularization universal in GCMs?

Yes. The following additions appear in every operational GCM/ESM without physical derivation from the governing equations:

- Horizontal hyperdiffusion of the form $(-1)^{n+1}K_n\nabla^{2n}$ applied to winds and temperature at every timestep. The order n and coefficient K_n are chosen for stability, not physics. In spectral models this is applied as a sharp spectral cutoff, effectively setting all modes above a prescribed wavenumber to zero.
- Vertical diffusion with coefficients tuned empirically.

- Sponge layers and relaxation nudging at lateral boundaries of all regional models (illegal fix for the Oliger-Sundström ill-posedness).
- Monotonicity-preserving flux limiters in tracer advection that clip all negative concentrations to zero (conservation-violating, non-differentiable).
- Convective adjustment schemes that instantaneously remove conditional instability, introducing finite jumps in the temperature profile.

Each of these modifies the mathematical character of the governing equations without physical justification. Each introduces a free parameter whose value affects model output at all scales through nonlinear interaction.

Has the chaos-ill-posedness interaction been investigated?

No. The two instability types are fundamentally different: chaos produces bounded Lyapunov divergence on the attractor; Hadamard ill-posedness produces unbounded growth at arbitrarily fine scales. When both operate simultaneously, the ill-posed short-wave modes generate spurious grid-scale energy that the chaotic nonlinear dynamics redistributes across all scales. This contamination is indistinguishable from physical variability in model output.

A direct, unexamined consequence: the CMIP multi-model spread is interpreted as representing structural uncertainty about the real climate system. But if different models suppress their ill-posed short-wave modes through different choices of hyperdiffusion order, sponge thickness, and limiter scheme — which they do — then part of the inter-model spread is artifact variability from differing levels of illegal regularization. This has never been quantified.

Have discontinuous parameterizations been analyzed mathematically?

No. The Jones-Prosperetti theorem , which guarantees bounded growth rates for algebraic source terms in two-phase models, requires Lipschitz continuity of the source vector $\mathbf{S}(\mathbf{q})$. The Picard-Lindelöf uniqueness theorem makes the same requirement.

Modern GCM parameterizations routinely violate Lipschitz continuity via:

- Convective triggering thresholds (convection on/off).
- Precipitation formation thresholds (rain/no-rain switch).
- Sea-ice albedo at the melting point (step function in temperature).
- Soil moisture thresholds at field capacity and wilting point.
- Richardson-number cutoffs in gravity wave drag.
- Any `IF/THEN` or `MAX/MIN` construct in parameterization code.

When the chaotic GCM trajectory repeatedly crosses a discontinuity surface at unpredictable times, a form of spurious stochastic behavior is produced that is: not physical (no process in nature has these mathematical properties); not properly statistical (it has no known probability distribution); and not convergent under grid refinement (finer grids generate more frequent and more abrupt threshold crossings).

This constitutes a third distinct source of model uncertainty — separate from initial-condition sensitivity (chaos) and parametric uncertainty — that has no name in the current GCM uncertainty literature.

Has the Climate Science Community Investigated These Issues?

Issue	State of knowledge	Operational status
Hydrostatic ill-posedness (lateral BCs)	Proved rigorously 1978; known to mathematicians	Not corrected: sponge layers, artificial viscosity used universally
Hydrostatic ill-posedness (Browning-Kreiss)	Proved 1986; alternative proposed but not adopted	Not corrected: global GCMs use hydrostatic core
Vertical discretization ill-posedness	Proved 2009	Not systematically checked in operational codes
Chaos-ill-posedness interaction	Never investigated	Absent from all GCM documentation and IPCC reports
Discontinuous parameterization analysis	Never formally analyzed	Thousands of if-then thresholds in all operational codes
CMIP spread from illegal regularization	Never quantified	Misattributed as structural uncertainty
Two-phase flow model ill-posedness	Thoroughly analyzed 1973–1984 by Lyczkowski et al., Ramshaw-Trapp, Stewart-Wendroff	Well-posed two-pressure models exist; not universally adopted in reactor safety codes until 1990s

The contrast between the nuclear reactor thermal-hydraulics community and the climate modeling community is stark and instructive. The reactor safety community, working under safety-critical accountability, spent the decade 1973–1984 rigorously cataloguing ill-posedness, developing well-posed hyperbolic alternatives, and formally verifying the mathematical structure of codes before accepting them for safety analysis .

The climate modeling community operates under different accountability pressures. Tuning and the addition of numerical fixes are treated as engineering necessities rarely reported in the scientific literature. The mathematical structure of GCM equation systems — whether they are well-posed, whether their parameterizations preserve hyperbolicity, whether their numerical schemes converge — is not regarded as a publishable or auditable question in the mainstream climate science literature.

This situation is difficult to justify given that GCM/ESM projections inform policy decisions of extraordinary economic and social consequence.

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